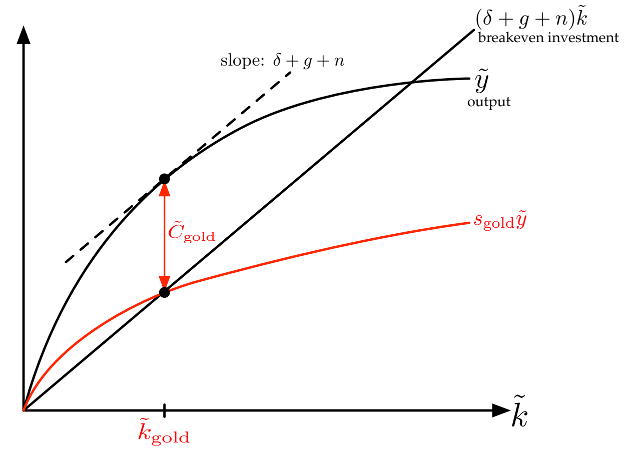
Handout 7

1. Suppose the production function is the following

$$Y = F(K, EL) = AK^{\alpha}(EL)^{1-\alpha} \tag{1}$$

Show that the **golden rule saving rate (s_{gold}) equals the capital share of income (α).**

Proof. As we can see from the graph below, we shift the breakeven investment curve to find the tangency point (Handout 4). So we will prove this by equating the slope of the production function and the breakeven investment curve.



First step, transform the production function to its intensive form:

$$\begin{aligned}\frac{Y}{EL} &= \frac{AK^{\alpha}(EL)^{1-\alpha}}{EL} \\ &= \frac{AK^{\alpha}}{(EL)^{\alpha}}\frac{(EL)^{1-\alpha}}{(EL)^{1-\alpha}} \\ &= A\tilde{k}^{\alpha}\end{aligned}$$

So we have our intensive form production function

$$\tilde{y} = A\tilde{k}^{\alpha}.$$

At the steady state corresponding to s_{gold}, the slope of the breakeven investment curve will be the same as the slope of the intensive form production function. So we have

$$\begin{aligned}&\frac{\partial \tilde{y}}{\partial \tilde{k}} = \delta+g+n \\ \Longrightarrow\ &\alpha A\tilde{k}^{\alpha-1} = \delta+g+n \\ \Longrightarrow\ &\alpha A\left(\frac{sA}{\delta+g+n}\right)^{\frac{\alpha-1}{1-\alpha}} = \delta+g+n \\ \Longrightarrow\ &\alpha A\left(\frac{\delta+g+n}{sA}\right) = \delta+g+n \\ \Longrightarrow\ &\alpha A = sA \\ \Longrightarrow\ &\boxed{\alpha = s}\ \text{ where } s \text{ is the golden rule saving rate.}\end{aligned}$$

■