Handout 1 (Math Review)

About Your TAs

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   - Office Hour: Wed. 11:00-1:00pm (only by appointment via `Calendly`)
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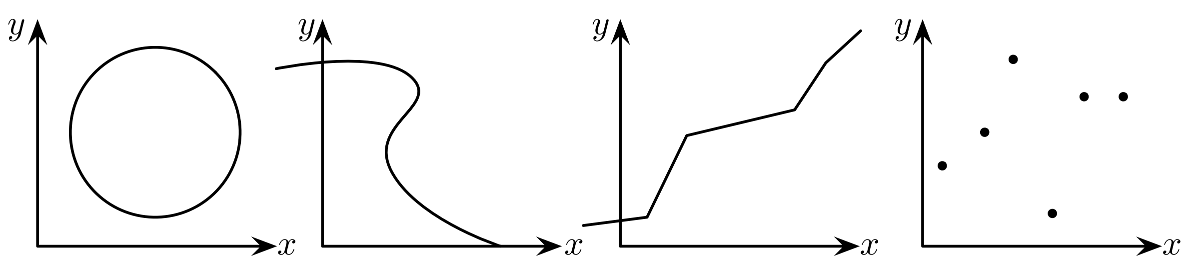
1 Logistics

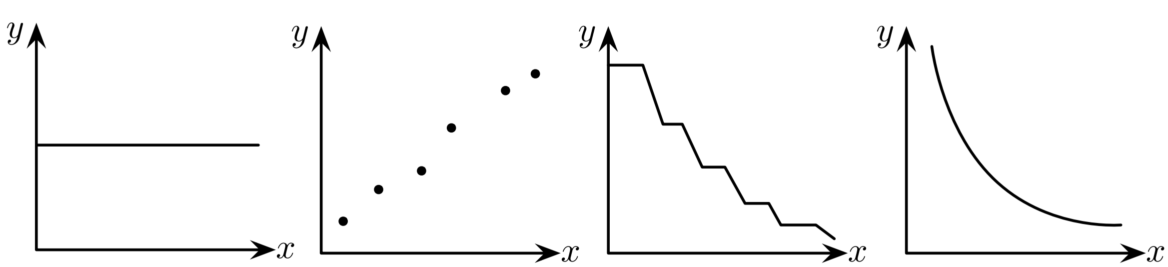
- Discussion sections will be conducted asynchronously.
- Handout: http://www.haochehsu.com (Handout can be found at the *Teaching* section)
- Register course Piazza.
- Any comments to us, please feel free to use the anonymous *Feedback Survey*.

2 Functions

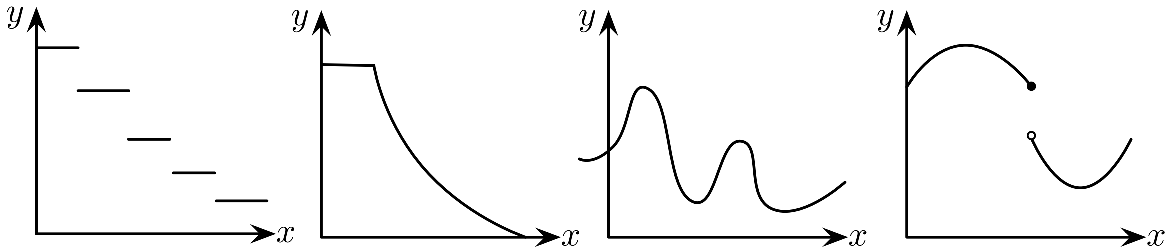
- Which graphs illustrates a **function**?



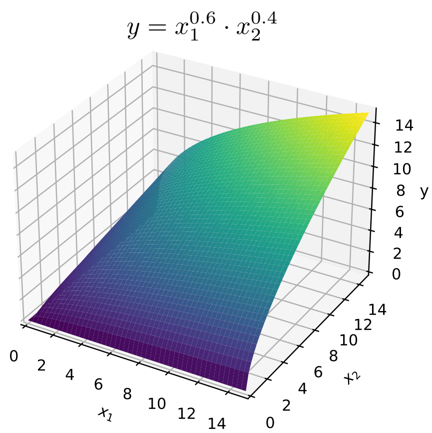
- Which graphs illustrates a **monotonic** function?

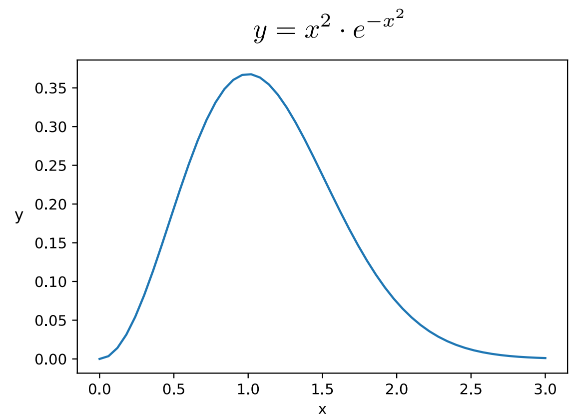


- Which graph illustrates a **smooth continuous** function?



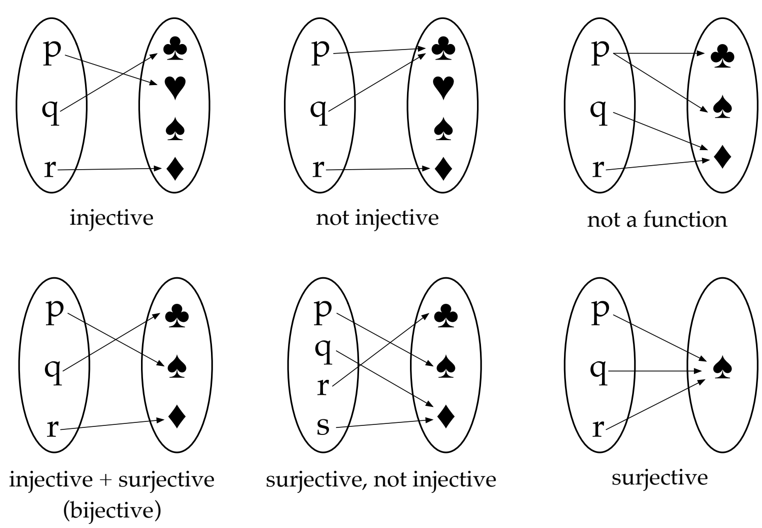
- The graphs of a **single variable** and **multi-variables** function





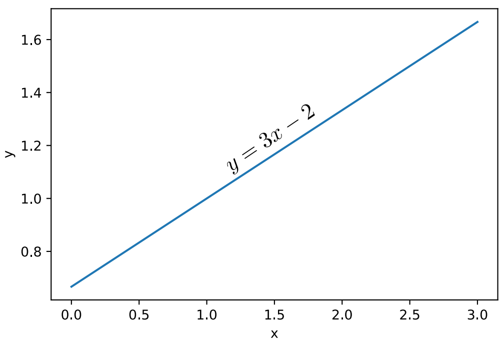
3 Function mappings and inversions

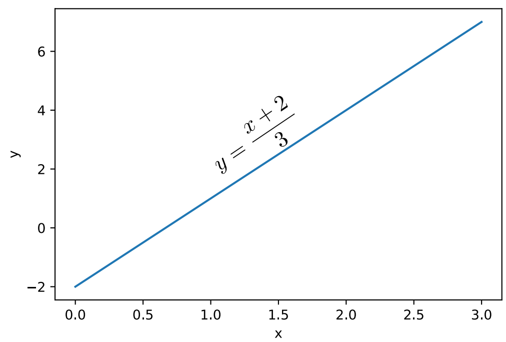
- A graphical representation of a function can be referred to figure 1 in the appendix.
- Mappings: **injective** (one-to-one), **surjective** (onto), **bijective** (one-to-one correspondence)



- An **inverse function** exists if and only if the function is **bijective**.

- The inverse function of $f(x) \equiv y = 3x - 2$





- Function inversion road map:
  1. Check whether the function is bijective or not.
  2. Switch x and y.
  3. Solve for y.
- Can we invert $y = x^2$?
- Find the inverse of $y = \frac{x+10}{4x-7}$.

4 Derivatives (If $f(x) = x^n$, then $f'(x) = n \cdot x^{n-1}$)

- Calculate the derivative of $y = x^2$.
- Calculate the derivative of $y = x^2 + \frac{1}{2}x + 3$.
- Calculate the derivative of $x^{10} + 3x = 2y + 4$.
- Calculate the derivative of $y = \left(x^2 + 7x + 2\right)^{\frac{1}{3}}$.
- Calculate the derivative of $y = x^2(9x + 2)$.
- Calculate the derivative of $y = \frac{x}{x^2+3}$.
- Calculate the derivative of $y = \sqrt{x^4 + 1}$.
- Calculate $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{1 + \sqrt{x^2 + 1}}$.
- Calculate the derivative of $y = 3e^{5x} - 8x^2$.
- Calculate the derivative of $y = x^3 \ln(x^2)$.

5 Line and Slope

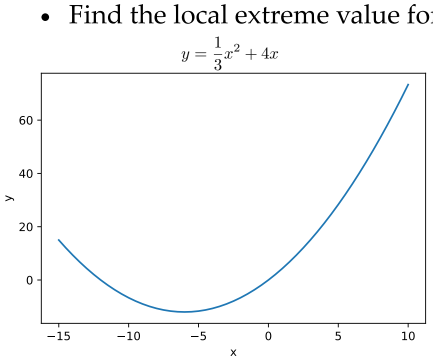
- Find the slope of a line passing through points $(-2, 4)$ and $(8, 9)$.
  - Roadmap:
    1. Set up the equation by similar triangles.
    2. Solve for y.
    3. Take derivative with respect to x.

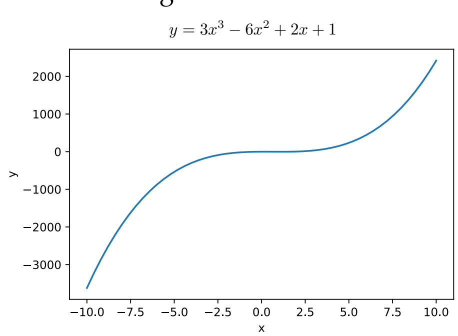
6 First order and second order derivatives

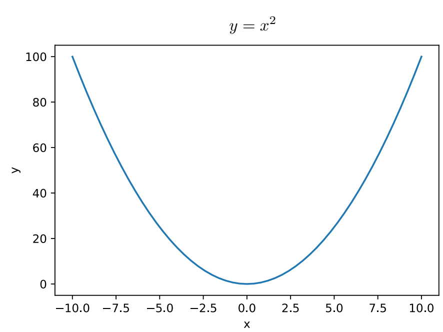
- First order derivation (F.O.C):
  - If $f'(p) > 0$, then $f(x)$ is an **increasing** function at point p.
  - If $f'(p) < 0$, then $f(x)$ is a **decreasing** function at point p.
  - If $f'(p) = 0$, point p is a **critical point** of $f(x)$.
- Second order derivation (S.O.C):
  - If $f''(p) > 0$, then $f(x)$ is a **convex** function (concave up) at point p.
  - If $f''(p) < 0$, then $f(x)$ is a **concave** function (concave down) at point p.
  - If $f''(p) = 0$, point p is an **inflation point** (point where the function changes concavity) of $f(x)$.

7 Unconstrained Optimization

- Local maximization and minimization road map:
  1. Take the first-order derivative.
  2. Set the derivative to 0 and solve for the value of x.
  3. Check the second-order derivative.
- Find the local extreme value for the following three cases:





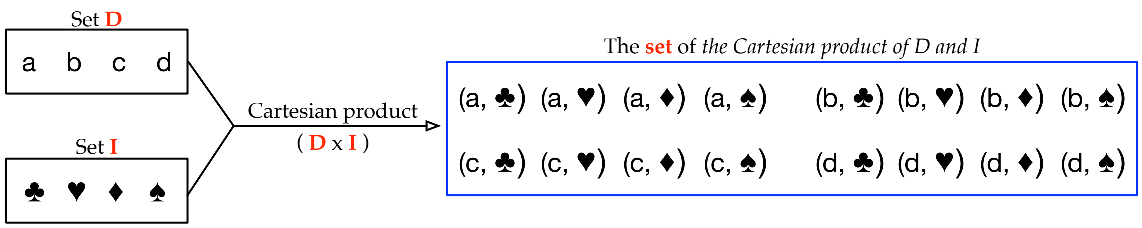


8 Constrained Optimization (optimization with binding equality)

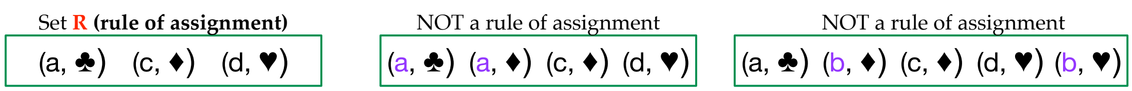
- Road map:
  1. Substitute the constraint into the objective function.
  2. Optimize the objective function by setting its first order condition to 0 and solve for parameters' optimal value.
  3. Plug the optimal values back to the objective function to obtain the optimized objective value.
- Minimize $f(x, y) = x^2 + y$ subject to $y = 3$.
- Maximize $f(x, y) = xy$ subject to $x + y = 6$.

Appendix: Set and Function

- A *set* is a collection of elements (objects).



- A *rule of assignment* is a **subset** (let's call it R) of the Cartesian product of two sets D and I so that each element of D appears at most one time as the first coordinate of the pair belonging to R.



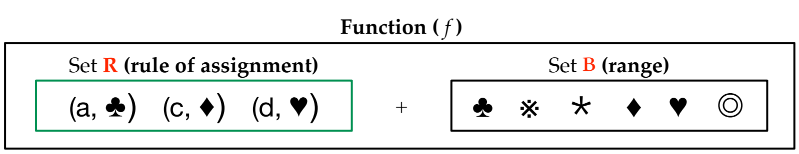
- There can be a lot of different *rule of assignments*.

- Now, lets **assign some names** to a *rule of assignment* (R). Let's look at the following *rule of assignment* example:

$$R = \{\ (a, ♣),\ (c, ♦),\ (d, ♥)\ \}\,.$$

  - The collection of the **first** coordinate of all pairs in R: $\{a, b, c\}$ is a set called **domain of R**.
  - The collection of the **second** coordinate of all pairs in R: $\{♣, ♦, ♥\}$ is a set called the **image set of R**.
  - For simplicity, we can drop the "of R" and just say the **domain** and the **image set** (we already know we are talking about the characteristics of R).

- A *function* (typically called f) is a *rule of assignment* (R) with a set B that contains the **image set of R**. We give a name for "set B," call it the **range of f**. To visualize a function, we can see the following:

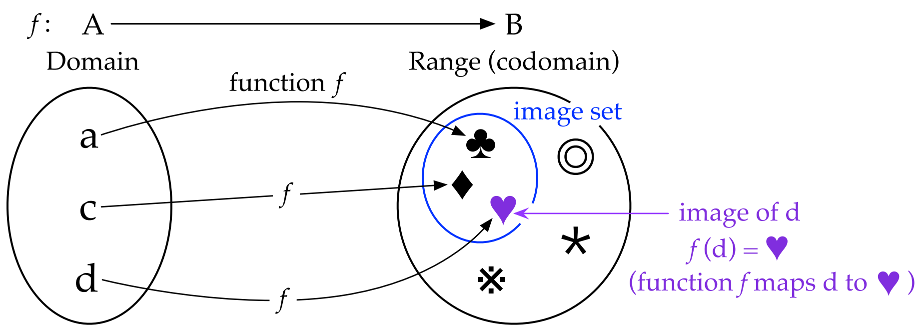


- So a function is a "two sets thing." One set is the **rule of assignment** which links or pairs an element from the **domain** to an element from the **image set.** The other is a set called **range** that contains all the elements in the image set and most of the time, includes some other elements that do not belongs to the image set.

- Let me break down the *rule of assignment* and emphasis the "linking" ability of a function and re-position the two sets and draw another graph that is **exactly the same** as above:

- The math representation of a function is $f : Domain \rightarrow Range$ or $f : A \rightarrow B$ where A is the domain and B is the range (codomain).

Figure 1: A graphical representation of a function



- This is the **correct** version! However, some people define things differently by referring the "set B" as "codomain" and call the "image set" the "range" and cause confusion. However, as you have the idea of the function in mind, people can call it whatever they want.