Week 9 Handout

1 Logistics

- HW on Chapter 20 is due on Monday, Dec. 7 at 11 a.m.
- Handout: http://www.haochehsu.com (Handout can be found at the *Teaching* section)
- Any comments to us, please feel free to use the anonymous *Feedback Survey*.

2 Profit Maximization

- A more abstract consideration of a firm's problem is

  $$\text{profit} = \text{revenues-costs}$$

  and revenue and cost are both functions of some arbitrary set of decisions a the firm makes; in that case, the firm's problem is

  $$\max_a \{R(a) - C(a)\}$$

  and the first-order condition with respect to any decision a_i is therefore

  $$\frac{\partial R}{\partial a_i} = \frac{\partial C}{\partial a_i}$$

  or, in Principles language, marginal revenue = marginal cost.

- But in our model, where firms are defined by their production function (their technological capabilities), the choices a firm makes is simply which production plan is feasible.

- We assume firms choose to maximize profits, which in our general model for a single-output, two-output firm is a very simple problem to state:

  $$\pi(p, w_1, w_2) = \max_{x_1, x_2} pf(x_1, x_2) - w_1 x_1 - w_2 x_2,$$

  where the solutions x^* are the profit-maximizing choices, and $f(x^*)$ is the output level.

- If we look at the FOC of maximizing π in this case, we get

  $$p\frac{\partial f(x)}{\partial x_i} = w_i$$

  which is exactly marginal revenue equals marginal cost.

- (This holds with equality if input x_i is used in a positive amount; if $x_i = 0$, it's possible that $p\frac{\partial f}{\partial x_i} < w_i$.)

- Combining the two FOCs for two inputs, we find

$$\frac{\partial f/\partial x_1}{\partial f/\partial x_2} = \frac{w_1}{w_2}$$

- So, like we said would happen, if the firm is maximizing profits and using inputs i and j, the Technical Rate of Substitution must be equal to the price ratio.
- (If it wasn't, the firm could change production slightly in a way that increased profits.)
- One thing to note: if the production function has constant or increasing returns to scale, then $\pi(p)$ is either zero or infinite.
- With CRS or IRS, if there were any feasible production plan with positive profit, we could just scale it up, and up, and up, since the production plan after scaling will still be feasible.
- So the only possibilities are infinite profits, negative or zero profits
- Negative profit can be ruled out if the firm can be shutdown to avoid additional costs.
- Infinite profits implies there is no solution to firm's problem.
- With CRS, there could still be positive production at zero profit.
- With DRS, there will be a unique solution for the firm's problem (since in this case the objective function is strictly concave).

Exercise

1. Consider the following production functions:

$$f(K, L) = K^{1/4} L^{1/2}$$

In the short run, K cannot be adjusted and it is 81.

a) Derive short run production function and short run profit function.

b) Find MPL. What is the optimal condition for labor demand?

c) Find optimal labor demand as a function of wage w and output price p.

d) Draw labor demand function with respect to the real wage $(\mathrm{w/P})$, when output price $\mathrm{P} = 1$.

3 Weak Axiom of Profit Maximization

- Assuming we only have 1 input now. Suppose we have observations

$$\left((p^1, w^1), (y^1, x^1)\right), \left((p^2, w^2), (y^2, x^2)\right), \ldots, \left((p^T, w^T), (y^T, x^T)\right)$$

of prices and production (net supply) choices

- Clearly, if the firm has a constant production function, it must contain all the points $\{((y^1, x^1), (y^2, x^2), \ldots, (y^T, x^T)\}$
- And if the firm was maximizing profits each time, it must be that for any t and s,

$$p^t y^t - w^t x^t \geq p^t y^s - w^t x^s$$

- This is called the Weak Axiom of Profit Maximization (WAPM)

Exercise

2. Consider the production of Irvine Dairy factory, and let's simplify its input to be cream and sugar, and the output is ice cream. Last week when prices of cream, sugar and ice cream were $(\$1, \$1, \$5)$, you learned from their production plan that it used 3 lbs of cream and 5 lbs of sugar to produce 2 gallon ice cream. This week the price of cream goes up by $\$1$ and the price of ice cream goes up by $\$1$ too. You don't know how much cream and sugar they use, but you do know that they produce 3 gallon ice cream. Suppose that you guess they use 3 lbs of cream and 10 lbs of sugar. Does this combination satisfies WAPM?